J. Phys. Chem. A **2003,** *107,* 695−705

Response Reactions: A Mathematical Well-Defined Way to Obtain Accurate Thermochemistry from ab Initio Calculations

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Received: September 27, 2002; In Final Form: November 24, 2002

The concept of stoichiometrically unique reactions referred to as response reactions (RERs), earlier deduced by us in applications of chemical thermodynamics and kinetics, is extended to isodesmic reactions. Namely, it is shown that the additional stoichiometric constraints according to which the number and type of chemical bonds in a multiple chemical reaction system are conserved may be formally appended to the conventional RERs formalism. As a result, one can define and generate a stoichiometrically unique class of isodesmic RERs. Along with the conventional RERs, the newly defined isodesmic RERs are shown to be effective in converting the total enthalpies of the species obtained from ab initio calculations into accurate values of the standard enthalpies of formations for large species.

1. Introduction

The term "isodesmic reaction" was introduced in 1970 by Hehre et al.[1] as a simple but effective means to partially cancel errors associated with incomplete basis sets and incomplete correction for electronic correlations. By definition,[2] an isodesmic reaction is a hypothetical chemical process in which the number and formal types of bonds are conserved in a reaction (i.e., remain the same on each side of the equation). Nowadays, the concept of isodesmic reactions is widely used not only in ab initio studies but as a general physicochemical tool for searching and quantitative estimation of the diverse stabilization/destabilization effects in organic molecules.[3,4] For instance, on the basis of isodesmic reactions, one can estimate the strain energy in cyclic compounds as compared with that of their acyclic analogues as models.[5−8]

It has been well established[9] that chemical stoichiometry does not provide any means of selecting a particular set of chemical reactions among a given set of species from an infinite number of possibilities. In other words, provided the only constraints among the species are the mass and charge balance conditions, chemical reactions may be generated arbitrarily and indefinitely. The additional constraint of conservation of the number and type of bonds in chemical reactions does not change this situation (i.e., isodesmic reactions can also be generated arbitrarily and indefinitely). It appears, however, that so far the fundamental problem of the stoichiometric uniqueness of isodesmic reactions has been overlooked. This may create serious difficulties that are next illustrated with the help of a simple example.

Consider the isodesmic reactions that have been recently proposed to estimate the strain energy of small ring molecules.[11] Bach and Dmitrenko[11] claim that they "expand upon the concept of group equivalent reactions that maintain, as closely as possible, the exact bonding environment" (i.e., the same number of C, H and O atoms bound to each atomic center) on both sides of the equation. In particular, for C_2H_4O (ethylene oxide), they cite the following reactions

$$C_2H_4O + 2CH_4 + H_2O = CH_3CH_3 + 2CH_3OH \tag{i}$$

$$C_2H_4O + CH_3CH_3 + 2CH_3OH = CH_3OCH_3 + 2CH_3CH_2OH \tag{ii}$$

Clearly, these reactions are not the only one that "maintain, as closely as possible, the exact bonding environment" of the ethylene oxide. Other examples are

$$C_2H_4O + 2CH_4 = CH_3CH_3 + CH_3OCH_3 \tag{iii}$$

$$C_2H_4O + CH_4 + H_2O = CH_3OH + CH_3CH_2OH \tag{iv}$$

As can be seen, we face the problem of stoichiometric arbitrariness of isodesmic reactions.

Most of the practitioners in the field seem to be well aware of the stoichiometric arbitrariness of isodesmic reactions. Thus, a large number of references may be given where a given species is described by a series of isodesmic reactions.[12−17] The problem is further substantially complicated by a lack of accurate experimental enthalpies of formation of many key species. Yet, the natural

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10.1021/jp027089a CCC: $25.00

question that can be discerned from the above brief discussion still remains: Which of the isodesmic reactions is the best? One of the possible ways to avoid the stoichiometric arbitrariness of the isodesmic reactions in obtaining an accurate estimation of the thermochemistry from ab initio calculations has been proposed by Raghavachari et al.[18] Namely, they proposed the use of the so-called "bond separation reactions" that are defined as isodesmic reactions involving the simplest parent molecules for every bond between non-hydrogen atoms. In particular, to validate the accuracy of ab initio methods, Raghavachari et al.[18] used a set of nine reference molecules: CH_4, NH_3, H_2O, $HC{\equiv}CH$, $H_2C{=}CH_2$, $H_3C{-}CH_3$, $H_2C{=}O$, $H_3C{-}OH$, and $H_3C{-}NH_2$.[19] An additional advantage of the bond separation reaction approach is that the standard enthalpies of formation of the reference species are usually known experimentally to an accuracy of ±0.4 kJ/mol (±0.1 kcal/mol). The bond separation reaction approach was later criticized[20] on the basis that the stoichiometric coefficients in the bond separation reactions may be very high so that even an accuracy of ±0.4 kJ/mol (±0.1 kcal/mol) for reference species may still result in large errors in the thermochemistry of larger molecules. Instead, Petersson et al[20] proposed to enlarge the number of reference species substantially, thus allowing the introduction of the so-called bond additivity corrections. The latter may be determined by fitting to the database of reference species. The theoretical background of this approach is the assumption that the errors in calculated bond energies are additive and that these errors are constant for each type of bond.

We have recently addressed the problem of the stoichiometric uniqueness of chemical reactions from thermodynamic and kinetic considerations. It appears that the basic thermodynamic and kinetic equations, when appropriately transformed, have the remarkable property to be partitioned into a sum of contributions associated with a finite and stoichiometrically unique set of reactions that were referred to as response reactions (RERs).[21] This unique stoichiometric partitioning reveals several completely new and fundamental features of multiple chemical reaction systems. Thus, using the RERs approach, we have been able to define the coupling among reactions adequately and the way to avoid it[21,22] to understand the unusual response of equilibrium multiple chemical reaction systems to different perturbations,[23,24] to deduce the theory of direct reaction routes from chemical thermodynamics and to develop a new method of their enumeration,[25,26] to establish the interrelationship between the theory of direct reaction routes and rate equations,[27] to model the microkinetics of the heterogeneous catalytic reactions,[27] and to formulate a fundamentally different approach to the generation of simplified reaction schemes.[28]

Here we extend the RERs approach to isodesmic reactions. More specifically, we show that the bond-preserving conditions may be easily incorporated into the RERs formalism. As a result, one can visualize and generate a special class of stoichiometrically unique isodesmic reactions (e.g., isodesmic RERs). Concomitantly, the RERs approach provides simple and effective algorithms for computer generation and enumeration of isodesmic reactions for any species starting from a given set of reference species of known composition and structure. Next, we apply the RERs formalism to convert the total energies of the species obtained from ab initio calculations into accurate thermochemistry for large molecules. The theoretical background of the application of RERs to extract accurate thermochemistry is similar to that employed by Petersson et al.[20] Namely, because of the stoichiometric uniqueness of RERs and because for a given species the RERs involve in a combinatorial manner all of the reference species, it may be anticipated that the errors due to the bond additivity may be mutually canceled when considering a sufficiently large number of reference species. Thus, by comparing the performance of the conventional and isodesmic RERs, one can check the validity and importance of the bond-preserving principle in ab initio methods. Finally, a conceptual synthesis of disjoint reasoning often provides new quantitative acumen and qualitative understanding. One such example is isoelectronic isogyric reactions that combine the "textbook" concept of isoelectronic species with the quantum chemical approach of isogyric reactions,[29] applied mostly for the understanding of the energetics of inorganic compounds.[30,31] Such is the joining of response and isodesmic reactions, which appears to be applicable to the study of strained and aromatic organic compounds alike.

2. Notation and Definitions

We consider a general multiple chemical reaction system comprising a set of n species $A_1, A_2, ..., A_n$. Each of the species in this system is characterized by a certain composition and structure. By composition, we mean a specified number of s "elements" B_1, $B_2, ..., B_s$ (e.g., any set of stoichiometrically atomic, molecular, or ionic entities that may be used to "construct" the species

$$A_i = \sum_{l=1}^{s} \beta_{li} B_l \tag{1}$$

These species need not be monatomic elements but may be component groups, as likewise understood in a linear algebraic formulation of chemical energetics.[32,33] Here β_{li} ($i = 1, 2, ..., n$; $l = 1, 2, ..., s$) is the number of elements B_l in the species A_i. The matrix

$$\boldsymbol{\beta} = \begin{bmatrix} \beta_{11} & \beta_{12} & ... & \beta_{1s} \\ \beta_{21} & \beta_{22} & ... & \beta_{2s} \\ ... & ... & ... & ... \\ \beta_{n1} & \beta_{n2} & ... & \beta_{ns} \end{bmatrix} \tag{2}$$

is normally referred to as the formula matrix.[9] From chemical stoichiometry, it is known that the elements $B_1, B_2, ..., B_s$ may always be selected so that the rank of the formula matrix be equal to the number of elements s. Therefore, in what follows we assume an appropriate selection of elements such that rank $\boldsymbol{\beta} = s$.

By the "structure" of the species we mean a specified type P_j ($j = 1, 2, ..., q$) and number π_{ij} ($i = 1, 2, ..., n$; $j = 1, 2, ..., q$) of bonds between the elements. Here, type of bond is mainly understood to be the classical covalent bond (i.e., single bond, double bond, etc.). Clearly, the term "type of bond" defined above is quite a rudimentary characteristic of the real structure of a molecule. More sophisticated types of bond may be defined and utilized. The mathematical formalism described in this work, however, is general and independent of the type of bond. Thus, we can define the matrix

$$\boldsymbol{\pi} = \begin{bmatrix} \pi_{11} & \pi_{12} & \dots & \pi_{1p} \\ \pi_{21} & \pi_{22} & \dots & \pi_{2p} \\ \dots & \dots & \dots & \dots \\ \pi_{n1} & \pi_{n2} & \dots & \pi_{np} \end{bmatrix} \tag{3}$$

that may be referred to as the bond matrix. The formula and bond matrices may be further combined into one formula−bond matrix

$$\mathbf{\Gamma}' = \begin{bmatrix} \beta_{11} & \beta_{12} & \dots & \beta_{1s} & \pi_{11} & \pi_{12} & \dots & \pi_{1p} \\ \beta_{21} & \beta_{22} & \dots & \beta_{2s} & \pi_{21} & \pi_{22} & \dots & \pi_{2p} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \beta_{n1} & \beta_{n2} & \dots & \beta_{ns} & \pi_{n1} & \pi_{n2} & \dots & \pi_{np} \end{bmatrix} \tag{4}$$

The formula−bond matrix is a quantitative characteristic of the composition and structure of a chemical system. Normally, the columns in this matrix are linearly dependent, that is, rank $\mathbf{\Gamma}' < s + p$. For our purposes, however, we need to consider only a linearly independent subset of columns from the formula−bond matrix. Without a loss of generality, we assume that these are the s columns from the formula matrix and the first q columns from the bond matrix. The remaining $p - q$ columns from the formula matrix may be omitted, thus defining a new, reduced formula−bond matrix

$$\mathbf{\Gamma} = \begin{bmatrix} \beta_{11} & \beta_{12} & \dots & \beta_{1s} & \pi_{11} & \pi_{12} & \dots & \pi_{1q} \\ \beta_{21} & \beta_{22} & \dots & \beta_{2s} & \pi_{21} & \pi_{22} & \dots & \pi_{2q} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \beta_{n1} & \beta_{n2} & \dots & \beta_{ns} & \pi_{n1} & \pi_{\mathrm{n}2} & \dots & \pi_{nq} \end{bmatrix} \tag{5}$$

in which the columns are linearly independent, that is, rank $\mathbf{\Gamma} = s + q$. Observe that the remaining $p - q$ columns from the formula−bond matrix may be presented as a linear combination of the first $s + q$ columns:

$$\begin{bmatrix} \pi_{1,k+h} \\ \pi_{2,k+h} \\ \dots \\ \pi_{n,k+h} \end{bmatrix} = \sum_{j=1}^{s} a_{jh} \begin{bmatrix} \beta_{1j} \\ \beta_{2j} \\ \dots \\ \beta_{nj} \end{bmatrix} + \sum_{j=1}^{k} b_{jh} \begin{bmatrix} \pi_{1j} \\ \pi_{2j} \\ \dots \\ \pi_{nj} \end{bmatrix} \qquad h = 1, 2, \dots, p - k \tag{6}$$

Here, a_{jh} and b_{jh} are constants.

Consider further a general chemical reaction r

$$r = \nu_1 \mathrm{A}_1 + \nu_2 \mathrm{A}_1 + \dots + \nu_n \mathrm{A}_n = 0 \tag{7}$$

where ν_i ($i = 1, 2, \dots, n$) are the stoichiometric coefficients that are assumed as usual to take positive values for products and negative values for reactants. Obviously, every chemical reaction satisfies the mass-balance conditions

$$\begin{aligned} &\nu_1\beta_{11} + \nu_2\beta_{21} + \dots + \nu_n\beta_{n1} = 0 \\ &\nu_1\beta_{12} + \nu_2\beta_{22} + \dots + \nu_n\beta_{n2} = 0 \\ &\dots \\ &\nu_1\beta_{1s} + \nu_2\beta_{2s} + \dots + \nu_n\beta_{ns} = 0 \end{aligned} \tag{8}$$

To avoid confusion in what follows, a reaction that satisfies only the mass-balance conditions is referred to as a conventional chemical reaction. It is well known[9] that in the absence of any other additional stoichiometric constraints the number of linearly independent conventional reactions is equal to $n -$ rank $\boldsymbol{\beta} = n - s$.

A reaction that additionally satisfies the mass-balance and bond-preserving conditions

$$\begin{aligned} &\nu_1\pi_{11} + \nu_2\pi_{21} + \dots + \nu_n\pi_{n1} = 0 \\ &\nu_1\pi_{12} + \nu_2\pi_{22} + \dots + \nu_n\pi_{n2} = 0 \\ &\dots \\ &\nu_1\pi_{1q} + \nu_2\pi_{2q} + \dots + \nu_n\pi_{nq} = 0 \end{aligned} \tag{9}$$

is called an isodesmic chemical reaction. Observe that eq 9 represents the bond-conservation condition for the first q types of bonds P_1, P_2,..., P_q. In view of eq 6, however, the bond-conservation conditions are valid for all p types of bonds. Clearly, the number of linearly independent isodesmic reactions is equal to $n -$ rank $\mathbf{\Gamma} = n - s - q$. A set of linearly independent isodesmic reactions may be derived using any appropriate linear algebra procedure, in particular, by inverting the formula−bond matrix.

3. Conventional Response Reactions

Definition: A reaction that involves no more than rank $\boldsymbol{\beta} + 1 = s + 1$ species is called a response reaction (RER).

Let the $s + 1$ species involved in a RER be $A_{i_1}, A_{i_2},..., A_{i_s}, A_{i_{s+1}}$, where $i_1, i_2,..., i_s, i_{s+1}$ is an ordered $(s + 1)$-tuple set of integers satisfying the condition $1 \le i_1 < i_2 < ... < i_s < i_{s+1} \le n$. A RER is denoted as $\rho(A_{i_1}, A_{i_2},..., A_{i_s}, A_{i_{s+1}})$, thus specifying the species that are involved in a RER. The general equation of a RER may be written as

$$\rho(A_{i_1}, A_{i_2}, ..., A_{i_s}, A_{i_{s+1}}) = \begin{vmatrix} \beta_{i_1,1} & \beta_{i_1,2} & ... & \beta_{i_1,s} & A_{i_1} \\ \beta_{i_2,1} & \beta_{i_2,2} & ... & \beta_{i_2,s} & A_{i_2} \\ ... & ... & ... & ... & ... \\ \beta_{i_s,1} & \beta_{i_s,2} & ... & \beta_{i_s,s} & A_{i_s} \\ \beta_{i_{s+1},1} & \beta_{i_{s+1},2} & ... & \beta_{i_{s+1},s} & A_{i_{s+1}} \end{vmatrix} = 0 \tag{10}$$

A complete list of RERs may be generated considering all of the possible combinations of $s + 1$ species $A_{i_1}, A_{i_2},..., A_{i_s}, A_{i_{s+1}}$ from a total of n species $A_1, A_2, ..., A_n$. That is, the total number N of RERs is equal to

$$N = \binom{n}{s+1} = \frac{n!}{(s+1)!(n-s-1)!}$$

Clearly, the number of RERs N exceeds the number of linearly independent RERs.

The enthalpy changes of the RERs $\Delta H(\rho) = \Delta H(A_{i_1}, A_{i_2},..., A_{i_s}, A_{i_{s+1}})$ may be expressed through the standard enthalpies of formation $\Delta_f H^\circ_{298}(A_{i_k})$ of the species via a similar formula

$$\Delta H(\rho) = \Delta H(A_{i_1}, A_{i_2},..., A_{i_s}, A_{i_{s+1}}) = \begin{vmatrix} \beta_{i_1,1} & \beta_{i_1,2} & ... & \beta_{i_1,s} & \Delta_f H^\circ_{298}(A_{i_1}) \\ \beta_{i_2,1} & \beta_{i_2,2} & ... & \beta_{i_2,s} & \Delta_f H^\circ_{298}(A_{i_2}) \\ ... & ... & ... & ... & ... \\ \beta_{i_s,1} & \beta_{i_s,2} & ... & \beta_{i_s,s} & \Delta_f H^\circ_{298}(A_{i_s}) \\ \beta_{i_{s+1},1} & \beta_{i_{s+1},2} & ... & \beta_{i_{s+1},s} & \Delta_f H^\circ_{298}(A_{i_{s+1}}) \end{vmatrix} \tag{11}$$

4. Isodesmic RERs

Definition: A reaction that is additionally subject to bond-preserving conditions and involves no more than rank $\Gamma + 1 = s + q + 1$ species is called an isodesmic RER.

Let $A_{j_1}, A_{j_2},..., A_{j_{s+q}}, A_{j_{s+q+1}} (1 \le j_1 < j_2 < ... < j_{s+q} < j_{s+q+1} \le n)$ be the $s + q + 1$ species involved in an isodesmic RER. Such a RER is denoted as η with the additional specification of the species involved in this RER (i.e., $\eta(A_{j_1}, A_{j_2},..., A_{j_{s+q}}, A_{j_{s+q+1}})$). The general equation of an isodesmic RER is

$$\eta(A_{i_1}, A_{i_2},..., A_{i_{s+q}}, A_{i_{s+q+1}}) = \begin{vmatrix} \beta_{j,1} & \beta_{j_1,2} & ... & \beta_{j_1,s} & \pi_{j,1} & \pi_{j_1,2} & ... & \pi_{j_1,q} & A_{j_1} \\ \beta_{j_2,1} & \beta_{j_2,2} & ... & \beta_{j_2,s} & \pi_{j_2,1} & \pi_{j_2,2} & ... & \pi_{j_2,q} & A_{j_2} \\ ... & ... & ... & ... & ... & ... & ... & ... & ... \\ \beta_{j_{s+q},1} & \beta_{i_{s+q},2} & ... & \beta_{j_{s+q},s} & \pi_{j_{s+q},1} & \pi_{j_{s+q},2} & ... & \pi_{j_{s+q},q} & A_{j_{s+q}} \\ \beta_{j_{s+q+1},1} & \beta_{j_{s+q+1},2} & ... & \beta_{j_{s+q+1},s} & \pi_{j_{s+q+1},1} & \pi_{j_{s+q+1},2} & ... & \pi_{j_{s+q+1},q} & A_{j_{s+q+1}} \end{vmatrix} = 0 \tag{12}$$

A complete list of isodesmic RERs may be generated by considering all of the possible choices of $q + 1$ species from a total of n. In other words, the total number N of isodesmic RERs does not exceed

$$N = \frac{n!}{(q+1)!(n-q-1)!}$$

Not necessarily all of the isodesmic RERs are stoichiometrically distinct. That is, like conventional RERs,[21] some of the stoichiometric coefficients in an isodesmic RER (or even all) may be equal to zero, thus resulting in stoichiometrically equivalent isostoichiometric RERs. Even so, the number of stoichiometrically distinct isodesmic RERs exceeds the number of linearly independent ones.

The enthalpy changes of the isodesmic RERs $\Delta H(\eta) = \Delta H(A_{j_1}, A_{j_2},..., A_{j_{s+q+1}})$ are interrelated with the standard enthalpies of formation $\Delta_f H^\circ_{298}(A_{i_k})$ of the species via

$$\Delta H(\eta) = \begin{vmatrix} \beta_{j,1} & \beta_{j_1,2} & ... & \beta_{j_1,s} & \pi_{j,1} & \pi_{j_1,2} & ... & \pi_{j_1,q} & \Delta_f H^\circ_{298}(A_{j_1}) \\ \beta_{j_2,1} & \beta_{j_2,2} & ... & \beta_{j_2,s} & \pi_{j_2,1} & \pi_{j_2,2} & ... & \pi_{j_2,q} & \Delta_f H^\circ_{298}(A_{j_2}) \\ ... & ... & ... & ... & ... & ... & ... & ... & ... \\ \beta_{j_{s+q},1} & \beta_{j_{s+q},2} & ... & \beta_{j_{s+q},s} & \pi_{j_{s+q},1} & \pi_{j_{s+q},2} & ... & \pi_{j_{s+q},q} & \Delta_f H^\circ_{298}(A_{j_{s+q}}) \\ \beta_{j_{s+q+1},1} & \beta_{j_{s+q+1},2} & ... & \beta_{j_{s+q+1},s} & \pi_{j_{s+q+1},1} & \pi_{j_{s+q+1},2} & ... & \pi_{j_{s+q+1},q} & \Delta_f H^\circ_{298}(A_{j_{s+q+1}}) \end{vmatrix} \tag{13}$$

5. RERs and Enthalpies of Formation of Species from ab Initio Calculations

Next, we briefly outline the RERs formalism in the evaluation of the standard enthalpy of formation of a given species, say A_1. Consider first the conventional RERs. Because the number of species involved in a conventional RER does not exceed $s + 1$ species, to enumerate a complete set of conventional RERs involving the species A_1, we need to specify only s species from the remaining $n - 1$. If these s species are A_{i_1}, A_{i_2},..., A_{i_s} ($2 \le i_1 < i_2 < ... < i_s < n$), then according to the above development the general equation of a conventional RER involving the species A_1 is given by

$$\rho(A_1, A_{i_1}, A_{i_2},..., A_{i_s}) = \begin{vmatrix} \beta_{11} & \beta_{12} & ... & \beta_{1s} & A_1 \\ \beta_{i_1,1} & \beta_{i_1,2} & ... & \beta_{i_1,s} & A_{i_1} \\ \beta_{i_2,1} & \beta_{i_2,2} & ... & \beta_{i_2,s} & A_{i_2} \\ ... & ... & ... & ... & ... \\ \beta_{i_s,1} & \beta_{i_s,2} & ... & \beta_{i_s,s} & A_{i_s} \end{vmatrix} = 0 \tag{14}$$

Obviously, the number of conventional RERs involving A_1 does not exceed the number of ways s species may be selected from a total of $n - 1$:

$$N = \frac{(n-1)!}{s!(n-s-1)!}$$

Now, let $H^\circ_{298}(A_i)$ ($i = 1, 2,..., n$) be the total enthalpies of the species at 298 K or even the total energies with or without zero-point or thermal corrections evaluated from ab initio calculations. It is further assumed that the standard enthalpies of formation of the species A_2, A_3,..., A_n are known from the experiment. Then, the (unknown) enthalpy of formation $\Delta_f H^\circ_{298}(A_1)$ of the species A_1 involved in a given conventional RER $\rho(A_1, A_{i_1}, A_{i_2},..., A_{i_s})$ may be evaluated by solving the following equation for $x = \Delta_f H^\circ_{298}(A_1)$:

$$\begin{vmatrix} \beta_{11} & \beta_{12} & ... & \beta_{1s} & x \\ \beta_{i_1,1} & \beta_{i_1,2} & ... & \beta_{i_1,s} & \Delta_f H^\circ_{298}(A_{i_1}) \\ \beta_{i_2,1} & \beta_{i_2,2} & ... & \beta_{i_2,s} & \Delta_f H^\circ_{298}(A_{i_2}) \\ ... & ... & ... & ... & ... \\ \beta_{i_s,1} & \beta_{i_s,2} & ... & \beta_{i_s,s} & \Delta_f H^\circ_{298}(A_{i_s}) \end{vmatrix} = \begin{vmatrix} \beta_{11} & \beta_{12} & ... & \beta_{1s} & H^\circ_{298}(A_1) \\ \beta_{i_1,1} & \beta_{i_1,2} & ... & \beta_{i_1,s} & H^\circ_{298}(A_{i_1}) \\ \beta_{i_2,1} & \beta_{i_2,2} & ... & \beta_{i_2,s} & H^\circ_{298}(A_{i_2}) \\ ... & ... & ... & ... & ... \\ \beta_{i_s,1} & \beta_{i_s,2} & ... & \beta_{i_s,s} & H^\circ_{298}(A_{i_s}) \end{vmatrix} \tag{15}$$

The final $\Delta_f H^\circ_{298}(A_1)$ is obtained as an average over a complete set of conventional RERs involving B_1.

Similarly, to estimate the enthalpy of formation of the species B_1 we start with a complete enumeration of the isodesmic RERs involving B_1:

$$\eta(A_1, A_{j_1}, A_{j_2},..., A_{j_{s+q}}) = \begin{vmatrix} \beta_{11} & \beta_{12} & ... & \beta_{1s} & \pi_{11} & \pi_{12} & ... & \pi_{1q} & A_1 \\ \beta_{j_1,1} & \beta_{j_1,2} & ... & \beta_{j_1,s} & \pi_{j_1,1} & \pi_{j_1,2} & ... & \pi_{j_1,q} & A_{j1} \\ \beta_{j_2,1} & \beta_{j_2,2} & ... & \beta_{j_2,s} & \pi_{j_2,1} & \pi_{j_2,2} & ... & \pi_{j_2,q} & A_{j_2} \\ ... & ... & ... & ... & ... & ... & ... & ... & ... \\ \beta_{j_{s+q},1} & \beta_{j_{s+q},2} & ... & \beta_{j_{s+q},s} & \pi_{j_{s+q},1} & \pi_{j_{s+q},2} & ... & \pi_{j_{s+q},q} & A_{j_{s+q}} \end{vmatrix} = 0 \tag{16}$$

Thus, the total number of isodesmic RERs involving A_1 does not exceed

$$N = \frac{(n-1)!}{(s+q)!(n-s-q-1)!}$$

For every isodesmic RER, the enthalpy of formation of A_1 is evaluated by solving the following equation for $x = \Delta_f H^\circ_{298}(A_1)$:

$$\begin{vmatrix} \beta_{11} & \beta_{12} & ... & \beta_{1s} & \pi_{11} & \pi_{12} & ... & \pi_{1q} & x \\ \beta_{j_1,1} & \beta_{j_1,2} & ... & \beta_{j_1,s} & \pi_{j_1,1} & \pi_{j_1,2} & ... & \pi_{j_1,q} & \Delta_f H^\circ_{298}(A_{j1}) \\ \beta_{j_2,1} & \beta_{j_2,2} & ... & \beta_{j_2,s} & \pi_{j_2,1} & \pi_{j_2,2} & ... & \pi_{j_2,q} & \Delta_f H^\circ_{298}(A_{j_2}) \\ ... & ... & ... & ... & ... & ... & ... & ... & ... \\ \beta_{j_{s+q},1} & \beta_{j_{s+q},2} & ... & \beta_{j_{s+q},s} & \pi_{j_{s+q},1} & \pi_{j_{s+q},2} & ... & \pi_{j_{s+q},q} & \Delta_f H^\circ_{298}(A_{j_{s+q}}) \end{vmatrix} = \begin{vmatrix} \beta_{11} & \beta_{12} & ... & \beta_{1s} & \pi_{11} & \pi_{12} & ... & \pi_{1q} & H^\circ_{298}(A_1) \\ \beta_{j_1,1} & \beta_{j_1,2} & ... & \beta_{j_1,s} & \pi_{j_1,1} & \pi_{j_1,2} & ... & \pi_{j_1,q} & H^\circ_{298}(A_{j_1}) \\ \beta_{j_2,1} & \beta_{j_2,2} & ... & \beta_{j_2,s} & \pi_{j_2,1} & \pi_{j_2,2} & ... & \pi_{j_2,q} & H^\circ_{298}(A_{j_2}) \\ ... & ... & ... & ... & ... & ... & ... & ... & ... \\ \beta_{j_{s+q},1} & \beta_{j_{s+q},2} & ... & \beta_{j_{s+q},s} & \pi_{j_{s+q},1} & \pi_{j_{s+q},2} & ... & \pi_{j_{s+q},q} & H^\circ_{298}(A_{j_{s+q}}) \end{vmatrix} \tag{17}$$

Repeating this procedure over a complete set of isodesmic RERs and taking the average results in the final value of the enthalpy of formation of the species A_1.

An Example. Consider the evaluation of the standard heat of formation of the ethylene oxide C_2H_4O based on ab initio calculations. The necessary data are presented in Table 1. Following Bach and Dmitrenko,[11] an appropriate system for consideration (i.e., a set

TABLE 1: Total Enthalpies (hartrees) at the G2 Level and the Experimental Standard Enthalpy of Formation (kJ/mol) of the Species Considered in This Work

considered species	$H^\circ_{298}(A_i)^a$	$\Delta_f H^\circ_{298}(A_i)^b$	considered species	$H^\circ_{298}(A_i)^a$	$\Delta_f H^\circ_{298}(A_i)^b$
1. cyclopropane − C_3H_6	−117.62679	53.1	5. cyclobutene − C_4H_6	−155.63905	157.0
2. ethylene oxide − C_2H_4O	−153.52877	−52.6	6. furan − C_4H_4O	−229.62792	−34.7
3. thiirane − C_2H_4S	−476.16012	82.3	7. benzene − C_6H_6	−231.77508	82.9
4. aziridine − C_2H_4NH	−133.65993	126.4			
reference species			reference species		
1. CH_4	−40.40707	−74.9	11. CH_3SCH_3	−477.36599	−37.6
2. H_2O	−76.32826	−241.8	12. CH_3NHCH_3	−134.87737	−18.4
3. H_2S	−398.92693	−20.5	13. CH_3CH_2OH	−154.75915	−235.3
4. NH_3	−56.45484	−45.9	14. CH_3CH_2SH	−477.36864	−46.2
5. CH_3CH_3	−79.62641	−83.8	15. $CH_3CH_2NH_2$	−134.88915	−47.3
6. $CH_2{=}CH_2$	−78.41192	52.5	16. $CH_3CH_2CH_3$	−118.85022	−104.6
7. CH_3OH	−115.53060	−201.0	17. $CH_2{=}CHCH_3$	−117.63998	20.4
8. CH_3SH	−438.14389	−22.8	18. $CH_3CH_2CH_2CH_3$	−158.07430	−127.1
9. CH_3NH_2	−95.66252	−23.0	19. $CH_2{=}CHCH_2{-}CH_3$	−156.86352	−0.5
10. CH_3OCH_3	−154.74133	−184.1	20. $CH_2{=}CHCH{=}CH_2$	−155.85855	110.0

[a] Data at the G2 level from NIST Computational Chemistry Comparison and the Benchmark Database (CCCBDB). [b] Afeefy, H. Y.; Liebman, J. F.; Stein, S. E. Neutral Thermochemical Data. In *NIST Chemistry WebBook*; Linstrom. P. J., Mallard, W. G., Eds.; NIST Standard Reference Database Number 69, National Institute of Standards and Technology: Gaithersburg, MD, 2002; (http://webbook.nist.gov).

of reference species) is {C_2H_4O, CH_4, H_2O, CH_3CH_3, CH_3OH, CH_3OCH_3, CH_3CH_2OH}. First, let us generate a complete set of conventional RERs involving C_2H_4O. For this system, we have from $s = 3$ elements (C, H, O). The rank of the formula matrix

$$\boldsymbol{\beta} = \begin{array}{ccc|l} C & H & O & \\ 2 & 4 & 1 & C_2H_4O \\ 1 & 4 & 0 & CH_4 \\ 0 & 2 & 1 & H_2O \\ 2 & 6 & 0 & CH_3CH_3 \\ 1 & 4 & 1 & CH_3OH \\ 2 & 6 & 1 & CH_3OCH_3 \\ 2 & 6 & 1 & CH_3CH_2OH \end{array}$$

is equal to 3; consequently, a conventional RER involves no more than 3 + 1 = 4 species. One of these species should always be C_2H_4O. The three other species should be selected in a combinatorial manner from the remaining six species. That is, the total number of conventional RERs involving C_2H_4O will not exceed 6!/3!/3! = 20. For instance, the species C_2H_4O, CH_4, H_2O, and CH_3CH_3 define the following conventional RER

$$\rho(C_2H_4O, CH_4, H_2O, CH_3CH_3) = \begin{vmatrix} 2 & 4 & 1 & C_2H_4O \\ 1 & 4 & 0 & CH_4 \\ 0 & 2 & 1 & H_2O \\ 2 & 6 & 0 & CH_3CH_3 \end{vmatrix}$$

$$= -2C_2H_4O - 8CH_4 + 2H_2O + 6CH_3CH_3 = 0$$

or in a more conventional way

$$C_2H_4O + 4CH_4 = 3CH_3CH_3 + H_2O$$

According to eq 15, the standard enthalpy of formation of C_2H_4O at the G2 level may be evaluated by solving the following equation for $x = \Delta_f H^\circ_{298}(C_2H_4O)$:

$$\begin{vmatrix} 2 & 4 & 1 & x \\ 1 & 4 & 0 & \Delta_f H^\circ_{298}(CH_4) \\ 0 & 2 & 1 & \Delta_f H^\circ_{298}(H_2O) \\ 2 & 6 & 0 & \Delta_f H^\circ_{298}(CH_3CH_3) \end{vmatrix} = \begin{vmatrix} 2 & 4 & 1 & H^\circ_{298}(C_2H_4O) \\ 1 & 4 & 0 & H^\circ_{298}(CH_4) \\ 0 & 2 & 1 & H^\circ_{298}(H_2O) \\ 2 & 6 & 0 & H^\circ_{298}(CH_3CH_3) \end{vmatrix}$$

or using the data given in Table 1

$$\begin{vmatrix} 2 & 4 & 1 & x \\ 1 & 4 & 0 & -74.9 \\ 0 & 2 & 1 & -241.8 \\ 2 & 6 & 0 & -83.8 \end{vmatrix} = \begin{vmatrix} 2 & 4 & 1 & -153.52877 \\ 1 & 4 & 0 & -40.40707 \\ 0 & 2 & 1 & -76.32826 \\ 2 & 6 & 0 & -79.62641 \end{vmatrix} 2625.5$$

Here 2625.5 is the factor converting hartrees into kJ/mol. The solution of this equation is $x = \Delta_f H^\circ_{298}(C_2H_4O) = -61.1$ kJ/mol. Continuing this procedure over a complete set of conventional RERs (Table 2) results in an average standard enthalpy of formation of C_2H_4O that is equal to −53.2 kJ/mol. This value is in good agreement with the experimental value of −52.6 kJ/mol.

Consider next the isodesmic RERs involving C_2H_4O. The species in our system involve four types of bonds, namely, C−C, C−O, H−O, and C−H. The extended formula−bond matrix is

$$\Gamma' = \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array}\begin{array}{ccccccc|l} \text{C} & \text{H} & \text{O} & \text{C-C} & \text{C-O} & \text{H-O} & \text{C-H} & \\ 2 & 4 & 1 & 1 & 2 & 0 & 4 & C_2H_4O \\ 1 & 4 & 0 & 0 & 0 & 0 & 4 & CH_4 \\ 0 & 2 & 1 & 0 & 0 & 2 & 0 & H_2O \\ 2 & 6 & 0 & 1 & 0 & 0 & 6 & CH_3CH_3 \\ 1 & 4 & 1 & 0 & 1 & 1 & 3 & CH_3OH \\ 2 & 6 & 1 & 0 & 2 & 0 & 8 & CH_3OCH_3 \\ 2 & 6 & 1 & 1 & 1 & 1 & 5 & CH_3CH_2OH \end{array}$$

The rank of this matrix is equal to 4, hence we can arbitrarily drop 3 columns. Assuming these to be the last three columns, for instance, results in a formula−bond matrix comprising $s + q = 3 + 1 = 4$ linearly independent columns

$$\Gamma = \begin{array}{cccc|l} \text{C} & \text{H} & \text{O} & \text{C-C} & \\ 2 & 4 & 1 & 1 & C_2H_4O \\ 1 & 4 & 0 & 0 & CH_4 \\ 0 & 2 & 1 & 0 & H_2O \\ 2 & 6 & 0 & 1 & CH_3CH_3 \\ 1 & 4 & 1 & 0 & CH_3OH \\ 2 & 6 & 1 & 0 & CH_3OCH_3 \\ 2 & 6 & 1 & 1 & CH_3CH_2OH \end{array}$$

Now, we are interested in enumerating a complete set of isodesmic RERs involving C_2H_4O. Because an isodesmic RER in this system involves no more than $s + q + 1 = 4 + 1 = 5$ species and 1 of these 5 species should always be C_2H_4O, we have to select only 4 species from the remaining 6. Thus, the number of isodesmic RERs involving C_2H_4O should not exceed 6!/2!/4! = 15. For example, the first 5 species define the following isodesmic RER

$$\eta(C_2H_4O, CH_4, H_2O, CH_3CH_3, CH_3OH) = \begin{vmatrix} 2 & 4 & 1 & 1 & C_2H_4O \\ 1 & 4 & 0 & 0 & CH_4 \\ 0 & 2 & 1 & 0 & H_2O \\ 2 & 6 & 0 & 1 & CH_3CH_3 \\ 1 & 4 & 1 & 0 & CH_3OH \end{vmatrix}$$

$$= -2C_2H_4O - 4CH_4 - 2H_2O + 2CH_3CH_3 + 4CH_3OH = 0$$

or in a more conventional form

$$C_2H_4O + 2CH_4 + H_2O = CH_3CH_3 + 2CH_3OH$$

According to eq 17, to determine the standard enthalpy of formation of C_2H_4O based on this isodesmic RER, we have to solve the following equation for $x = \Delta_f H_{298}{}^\circ(C_2H_4O)$

$$\begin{vmatrix} 2 & 4 & 1 & 1 & x \\ 1 & 4 & 0 & 0 & \Delta_f H^\circ_{298}(CH_4) \\ 0 & 2 & 1 & 0 & \Delta_f H^\circ_{298}(H_2O) \\ 2 & 6 & 0 & 1 & \Delta_f H^\circ_{298}(CH_3CH_3) \\ 1 & 4 & 1 & 0 & \Delta_f H^\circ_{298}(CH_3OH) \end{vmatrix} = \begin{vmatrix} 2 & 4 & 1 & 1 & H^\circ_{298}(C_2H_4O) \\ 1 & 4 & 0 & 0 & H^\circ_{298}(CH_4) \\ 0 & 2 & 1 & 0 & H^\circ_{298}(H_2O) \\ 2 & 6 & 0 & 1 & H^\circ_{298}(CH_3CH_3) \\ 1 & 4 & 1 & 0 & H^\circ_{298}(CH_3OH) \end{vmatrix}$$

or after inserting the data from Table 1

$$\begin{vmatrix} 2 & 4 & 1 & 1 & x \\ 1 & 4 & 0 & 0 & -74.9 \\ 0 & 2 & 1 & 0 & -241.8 \\ 2 & 6 & 0 & 1 & -83.8 \\ 1 & 4 & 1 & 0 & -201.0 \end{vmatrix} = \begin{vmatrix} 2 & 4 & 1 & 1 & -153.52877 \\ 1 & 4 & 0 & 0 & -40.40707 \\ 0 & 2 & 1 & 0 & -76.32826 \\ 2 & 6 & 0 & 1 & -79.62641 \\ 1 & 4 & 1 & 0 & -115.53060 \end{vmatrix} 2625.5$$

Solving this equation, we obtain $x = \Delta_f H^\circ_{298}(C_2H_4O) = -51.0$ kJ. The results of similar calculations for the remaining isodesmic RERs are summarized in Table 2. As can be seen, from a total of 15 possible isodesmic RERs involving C_2H_4O, only 7 are

TABLE 2: Complete Set of Stoichiometrically Distinct Conventional and Isodesmic RERs Involving Cycloprpane, Ethylene Oxide, Thiirane, Aziridine, Cyclobutene, Furan, and Benzene[a]

(1) Cyclopropane − C_3H_6

(a) Simultaneously Conventional and Isodesmic RERs

	$\Delta_f H^\circ_{298}(A_i)$		$\Delta_f H^\circ_{298}(A_i)$
1. $\mathbf{C_3H_6} + 3CH_4 = 3CH_3CH_3$	55.3	3. $\mathbf{C_3H_6} + 3CH_3CH_3 = 3CH_3CH_2CH_3$	54.5
2. $\mathbf{C_3H_6} + 3/2CH_4 = 3/2CH_3CH_2CH_3$	55.0	average:	54.9
		std dev	0.4

(2) Ethylene Oxide − C_2H_4O

(a) Conventional RERs

	$\Delta_f H^\circ_{298}(A_i)$		$\Delta_f H^\circ_{298}(A_i)$
1. $\mathbf{C_2H_4O} + 4CH_4 = 3\ CH_3CH_3 + H_2O$	−61.1	9. $\mathbf{C_2H_4O} + CH_4 + CH_3OH = 2CH_3CH_2OH$	−58.5
2. $\mathbf{C_2H_4O} + CH_4 + 1/2H_2O = 3/2CH_3OCH_3$	−48.8	10. $\mathbf{C_2H_4O} + C_2H_6 + H_2O = 2CH_3OCH_3$	−44.7
3. $\mathbf{C_2H_4O} + CH_4 + 1/2H_2O = 3/2CH_3CH_2OH$	−55.4	11. $\mathbf{C_2H_4O} + CH_3CH_3 + H_2O = 2CH_3CH_2OH$	−55.3
4. $\mathbf{C_2H_4O} + CH_4 + 2H_2O = 3CH_3OH$	−46.1	12. $\mathbf{C_2H_4O} + CH_3CH_3 + 2CH_3OH = 3CH_3OCH_3$	−46.5
5. $\mathbf{C_2H_4O} + 2CH_4 = CH_3CH_3 + CH_3OCH_3$	−52.9	13. $\mathbf{C_2H_4O} + CH_3CH_3 + 2CH_3OH = 3CH_3CH_2OH$	−59.8
6. $\mathbf{C_2H_4O} + 2CH_4 = CH_3CH_3 + CH_3CH_2OH$	−57.3	average:	−53.2
7. $\mathbf{C_2H_4O} + 3CH_4 = 2CH_3CH_3 + CH_3OH$	−56.1	std dev	5.5
8. $\mathbf{C_2H_4O} + CH_4 + CH_3OH = 2CH_3OCH_3$	−49.7		

(b) Isodemic RERs

	$\Delta_f H^\circ_{298}(A_i)$		$(\Delta_f H^\circ_{298}(A_i))$
1. $\mathbf{C_2H_4O} + 2CH_4 + H_2O = CH_3CH_3 + 2CH_3OH$	−51.0	6. $\mathbf{C_2H_4O} + CH_4 + CH_3OH = CH_3OCH_3 + CH_3CH_2OH$	−54.1
2. $\mathbf{C_2H_4O} + 2CH_4 = CH_3CH_3 + CH_3OCH_3$	−52.9	7. $\mathbf{C_2H_4O} + CH_3CH_3 + 2CH_3OH = CH_3OCH_3 + 2CH_3CH_2OH$	−55.3
3. $\mathbf{C_2H_4O} + CH_3CH_3 + H_2O = 2CH_3CH_2OH$	−53.5	average:	−53.2
4. $\mathbf{C_2H_4O} + CH_4 + H_2O = CH_3OH + CH_3CH_2OH$	−52.3	std dev	1.4
5. $\mathbf{C_2H_4O} + CH_4 + 1/2H_2O = 1/2CH_3OCH_3 + CH_3CH_2OH$	−53.2		

(3) Thiirane − C_2H_4S

(a) Conventional RERs

	$\Delta_f H^\circ_{298}(A_i)$		$\Delta_f H^\circ_{298}(A_i)$
1. $\mathbf{C_2H_4S} + 4CH_4 = 3\ CH_3CH_3 + H_2S$	74.9	9. $\mathbf{C_2H_4S} + CH_4 + CH_3SH = 2CH_3CH_2SH$	74.1
2. $\mathbf{C_2H_4S} + CH_4 + 1/2H_2S = 3/2CH_3SCH_3$	76.9	10. $\mathbf{C_2H_4S} + CH_3CH_3 + H_2S = 2CH_3SCH_3$	77.5
3. $\mathbf{C_2H_4S} + CH_4 + 1/2H_2S = 3/2CH_3CH_2SH$	74.4	11. $\mathbf{C_2H_4S} + CH_3CH_3 + H_2S = 2CH_3CH_2SH$	74.2
4. $\mathbf{C_2H_4S} + CH_4 + 2H_2S = 3CH_3SH$	75.4	12. $\mathbf{C_2H_4S} + CH_3CH_3 + 2CH_3SH = 3CH_3SCH_3$	78.5
5. $\mathbf{C_2H_4S} + 2CH_4 = CH_3CH_3 + CH_3SCH_3$	76.2	13. $\mathbf{C_2H_4S} + CH_3CH_3 + 2CH_3SH = 3CH_3CH_2SH$	73.6
6. $\mathbf{C_2H_4S} + 2CH_4 = CH_3CH_3 + CH_3CH_2SH$	74.6	average:	75.6
7. $\mathbf{C_2H_4S} + 3CH_4 = 2\ CH_3CH_3 + CH_3SH$	75.1	std dev	1.5
8. $\mathbf{C_2H_4S} + CH_4 + CH_3SH = 2CH_3SCH_3$	77.4		

(b) Isodesmic RERs

	$\Delta_f H^\circ_{298}(A_i)$		$\Delta_f H^\circ_{298}(A_i)$
1. $\mathbf{C_2H_4S} + 2CH_4 + H_2S = CH_3CH_3 + 2CH_3SH$	75.0	6. $\mathbf{C_2H_4S} + CH_4 + CH_3SH = CH_3SCH_3 + CH_3CH_2SH$	75.7
2. $\mathbf{C_2H_4S} + 2CH_4 = CH_3CH_3 + CH_3SCH_3$	76.0	7. $\mathbf{C_2H_4S} + CH_3CH_3 + 2CH_3SH = CH_3SCH_3 + 2CH_3CH_2SH$	75.4
3. $\mathbf{C_2H_4S} + CH_3CH_3 + H_2S = 2CH_3CH_2SH$	74.4	average:	75.2
4. $\mathbf{C_2H_4S} + CH_4 + H_2S = CH_3SH + CH_3CH_2SH$	74.7	std dev	0.6
5. $\mathbf{C_2H_4S} + CH_4 + 1/2H_2S = 1/2CH_3SCH_3 + CH_3CH_2SH$	75.2		

(4) Aziridine − C_2H_4NH

(a) Conventional RERs

	$\Delta_f H^\circ_{29\,8}(A\ _i)$
1. $\mathbf{C_2H_4NH} + 4CH_4 = 3CH_3CH_3 + NH_3$	122.7
2. $\mathbf{C_2H_4NH} + CH_4 + 1/2NH_3 = 3/2CH_3NHCH_3$	127.1
3. $\mathbf{C_2H_4NH} + CH_4 + 1/2NH_3 = 3/2CH_3CH_2NH_2$	130.1
4. $\mathbf{C_2H_4NH} + CH_4 + 2NH_3 = 3CH_3NH_2$	126.3
5. $\mathbf{C_2H_4NH} + 2CH_4 = CH_3CH_3 + CH_3NHCH_3$	125.6
6. $\mathbf{C_2H_4NH} + 2CH_4 = CH_3CH_3 + CH_3CH_2NH_2$	127.6
7. $\mathbf{C_2H_4NH} + 3CH_4 = 2CH_3CH_3 + CH_3NH_2$	123.9
8. $\mathbf{C_2H_4NH} + CH_4 + CH_3NH_2 = 2CH_3NHCH_3$	127.3
9. $\mathbf{C_2H_4NH} + CH_4 + CH_3NH_2 = 2CH_3CH_2NH_2$	131.4
10. $\mathbf{C_2H_4NH} + CH_3CH_3 + NH_3 = 2CH_3NHCH_3$	128.5
11. $\mathbf{C_2H_4NH} + CH_3CH_3 + NH_3 = 2CH_3CH_2NH_2$	132.6
12. $\mathbf{C_2H_4NH} + CH_3CH_3 + 2CH_3NH_2 = 3CH_3NHCH_3$	129.0
13. $\mathbf{C_2H_4NH} + CH_3CH_3 + 2CH_3NH_2 = 3CH_3CH_2NH_2$	135.1
average:	128.3
std dev	3.5

(a) Isodesmic RERs

	$\Delta_f H^\circ_{298}(A_i)$
1. $\mathbf{C_2H_4NH} + 2CH_4 + NH_3 = CH_3CH_3 + 2CH_3NH_2$	125.1
2. $\mathbf{C_2H_4NH} + 2CH_4 = CH_3CH_3 + CH_3NHCH_3$	125.6
3. $\mathbf{C_2H_4NH} + CH_3CH_3 + NH_3 = 2CH_3CH_2NH_2$	132.6
4. $\mathbf{C_2H_4NH} + CH_4 + NH_3 = CH_3NH_2 + CH_3CH_2NH_2$	128.8
5. $\mathbf{C_2H_4NH} + CH_4 + 1/2NH_3 = 1/2CH_3NHCH_3 + CH_3CH_2NH_2$	129.1
6. $\mathbf{C_2H_4NH} + CH_4 + CH_3NH_2 = CH_3NHCH_3 + CH_3CH_2NH_2$	129.3
7. $\mathbf{C_2H_4NH} + CH_3CH_3 + 2CH_3NH_2 = CH_3NHCH_3 + 2CH_3CH_2NH_2$	133.1
average:	129.1
std dev	3.1

(5) Cyclobutene − C_4H_6

(a) Conventional RERs

	$\Delta_f H^\circ_{298}(A_i)$
1. $\mathbf{C_4H_6} + 6CH_4 = 5CH_3CH_3$	163.2
2. $\mathbf{C_4H_6} + CH_4 = 5/2CH_2{=}CH_2$	163.3
3. $\mathbf{C_4H_6} + 7/2CH_4 = 5/2CH_3CH_2CH_3$	162.5
4. $\mathbf{C_4H_6} + CH_4 = 5/3CH_2{=}CHCH_3$	162.8
5. $\mathbf{C_4H_6} + 8/3CH_4 = 5/3CH_3CH_2CH_2CH_3$	161.0
6. $\mathbf{C_4H_6} + CH_4 = 5/4CH_2{=}CHCH_2CH_3$	161.7
7. $\mathbf{C_4H_6} + CH_3CH_3 = 3CH_2{=}CH_2$	163.3
8. $\mathbf{C_4H_6} + 7CH_3CH_3 = 6CH_3CH_2CH_3$	161.6
9. $\mathbf{C_4H_6} + 2CH_3CH_3 = 2CH_2{=}CHCH_3$	162.7
10. $\mathbf{C_4H_6} + 4CH_3CH_3 = 3CH_3CH_2CH_2CH_3$	159.2
11. $\mathbf{C_4H_6} + CH_3CH_3 = 3/2CH_2{=}CHCH_2CH_3$	161.3
12. $\mathbf{C_4H_6} + CH_3CH_2CH_3 = 7/2CH_2{=}CH_2$	163.6
13. $\mathbf{C_4H_6} + CH_3CH_2CH_2CH_3 = 4CH_2{=}CH_2$	164.7
14. $\mathbf{C_4H_6} + CH_3CH_2CH_3 = 7/3CH_2{=}CHCH_3$	162.8
15. $\mathbf{C_4H_6} + 8CH_3CH_2CH_3 = 7CH_3CH_2CH_2CH_3$	156.1
16. $\mathbf{C_4H_6} + CH_3CH_2CH_3 = 7/4CH_2{=}CHCH_2CH_3$	161.3
17. $\mathbf{C_4H_6} + CH_3CH_2CH_2CH_3 = 8/3CH_2{=}CHCH_3$	163.8
18. $\mathbf{C_4H_6} + CH_3CH_2CH_2CH_3 = 2CH_2{=}CHCH_2CH_3$	162.0
average:	162.1
std dev	2.0

(b) Isodesmic RERs

	$\Delta_f H^\circ_{298}(A_i)$
1. $\mathbf{C_4H_6} + 4CH_4 = 3CH_3CH_3 + CH_2{=}CH_2$	163.2
2. $\mathbf{C_4H_6} + 3CH_4 = 2CH_3CH_3 + CH_2{=}CHCH_3$	162.9
3. $\mathbf{C_4H_6} + 5/2CH_4 = CH_2{=}CH_2 + 3/2CH_3CH_2CH_3$	162.8
4. $\mathbf{C_4H_6} + CH_4 + 2CH_2{=}CH_2 = 3CH_2{=}CHCH_3$	162.4
5. $\mathbf{C_4H_6} + 2CH_4 = CH_3CH_2CH_3 + CH_2{=}CHCH_3$	162.7
6. $\mathbf{C_4H_6} + 5CH_3CH_3 = CH_2{=}CH_2 + 4CH_3CH_2CH_3$	162.2
7. $\mathbf{C_4H_6} + CH_3CH_3 + 3CH_2{=}CH_2 = 4CH_2{=}CHCH_3$	162.0
8. $\mathbf{C_4H_6} + 4CH_3CH_3 = 3CH_3CH_2CH_3 + CH_2{=}CHCH_3$	162.1
9. $\mathbf{C_4H_6} + 4CH_3CH_3 + CH_3CH_2CH_3 = 5CH_2{=}CHCH_3$	162.0
10. $\mathbf{C_4H_6} + 2CH_4 = CH_2{=}CH_2 + CH_3CH_2CH_2CH_3$	161.9
11. $\mathbf{C_4H_6} + 5/3CH_4 = CH_2{=}CH{-}CH_3 + 2/3CH_3CH_2CH_2CH_3$	162.1
12. $\mathbf{C_4H_6} + 3CH_3CH_3 = CH_2{=}CH_2 + 2CH_3CH_2CH_2CH_3$	160.6
13. $\mathbf{C_4H_6} + 5/2CH_3CH_3 = CH_2{=}CH{-}CH_3 + 3/2CH_3CH_2CH_2CH_3$	161.0
14. $\mathbf{C_4H_6} + 6CH_3CH_2CH_3 = CH_2{=}CH_2 + 5CH_3CH_2CH_2CH_3$	158.2
15. $\mathbf{C_4H_6} + 5CH_2{=}CH_2 + CH_3CH_2CH_2CH_3 = 6CH_2{=}CHCH_3$	162.7
16. $\mathbf{C_4H_6} + 5CH_3CH_2CH_3 = 4CH_2{=}CHCH_3 + 4CH_3CH_2CH_2CH_3$	159.0
17. $\mathbf{C_4H_6} + 2CH_4 = CH_3CH_3 + CH_2{=}CHCH_2CH_3$	162.0
18. $\mathbf{C_4H_6} + CH_4 + 1/2CH_2{=}CH_2 = 3/2CH{=}CHCH_2{=}CH_3$	161.3
19. $\mathbf{C_4H_6} + 3/2CH_4 = 1/2CH_3CH_2CH_3 + CH_2{=}CHCH_2CH_3$	161.8
20. $\mathbf{C_4H_6} + CH_4 + CH_2{=}CHCH_3 = 2CH_2{=}CHCH_2CH_3$	161.0
21. $\mathbf{C_4H_6} + 4/3CH_4 = 1/3CH_3CH_2CH_2CH_3 + CH_2{=}CHCH_2CH_3$	161.5
22. $\mathbf{C_4H_6} + CH_3CH_3 + CH_2{=}CH_2 = 2CH_2{=}CHCH_2CH_3$	160.7
23. $\mathbf{C_4H_6} + 3CH_3CH_3 = 2CH_3CH_2CH_3 + CH_2{=}CHCH_2CH_3$	161.4
24. $\mathbf{C_4H_6} + CH_3CH_3 + 3CH_2{=}CHCH_3 = 3CH_2{=}CHCH_2CH_3$	160.0
25. $\mathbf{C_4H_6} + 2CH_3CH_3 = CH_3CH_2CH_2CH_3 + CH_2{=}CHCH_2CH_3$	160.6
26. $\mathbf{C_4H_6} + CH_3CH_2CH_3 + 3/2CH_2{=}CH_2 = 5/2CH_2{=}CHCH_2CH_3$	160.3
27. $\mathbf{C_4H_6} + 2CH_2{=}CH_2 + CH_3CH_2CH_2CH_3 = 3CH_2{=}CHCH_2CH_3$	160.7
28. $\mathbf{C_4H_6} + 3CH_2{=}CHCH_3 + CH_3CH_2CH_3 = 4CH_2{=}CHCH_2CH_3$	159.3
29. $\mathbf{C_4H_6} + 4CH_2{=}CHCH_3 = 3CH_3CH_2CH_2CH_3 + CH_2{=}CHCH_2CH_3$	159.1
30. $\mathbf{C_4H_6} + 4CH_2{=}CHCH_3 + CH_3CH_2CH_2CH_3 = 5CH_2{=}CHCH_2CH_3$	159.4
average:	161.2
std dev	1.3

TABLE 2 (Continued)

(6) Furan − C_4H_4O

(a) Conventional RERs

	$\Delta_f H^\circ_{298}(A_i)$
1. $\mathbf{C_4H_4O} + 10CH_4 = 7CH_3CH_3 + H_2O$	−41.3
2. $\mathbf{C_4H_4O} + 3CH_4 + 6H_2O = 7CH_3OH$	−5.8
3. $\mathbf{C_4H_4O} + 3CH_4 = 7/2CH_2{=}CH_2 + H_2O$	−41.2
4. $\mathbf{C_4H_4O} + 4CH_4 + 5H_2O = 7/2CH_3OCH_3$	−12.4
5. $\mathbf{C_4H_4O} + 9CH_4 = CH_3OH + 6CH_3CH_3$	−36.2
6. $\mathbf{C_4H_4O} + 8CH_4 = CH_3OCH_3 + 5CH_3CH_3$	−33.0
7. $\mathbf{C_4H_4O} + 3CH_4 = CH_3OH + 3CH_2{=}CH_2$	−36.1
8. $\mathbf{C_4H_4O} + 3CH_4 + 5CH_3OH = 6CH_3OCH_3$	−17.1
9. $\mathbf{C_4H_4O} + 3CH_4 = 5/2CH_3CH_3 + CH_3OCH_3$	−32.9
10. $\mathbf{C_4H_4O} + 3\ CH_3CH_3 + 8H_2O = 10CH_3OH$	9.4
11. $\mathbf{C_4H_4O} + 3CH_3CH_3 = 5CH_2{=}CH_2 + H_2O$	−41.1
12. $\mathbf{C_4H_4O} + 3CH_3CH_3 + 4H_2O = 5CH_3OCH_3$	0.0
13. $\mathbf{C_4H_4O} + 3CH_3CH_3 = CH_3OH + 9/2CH_2{=}CH_2$	−36.1
14. $\mathbf{C_4H_4O} + 8CH_3OH + 3CH_3CH_3 = 9CH_3OCH_3$	−7.5
15. $\mathbf{C_4H_4O} + 3CH_3CH_3 = 4CH_2{=}CH_2 + CH_3OCH_3$	−32.9
16. $\mathbf{C_4H_4O} + 8/5CH_4 = 7/5CH_2{=}CHCH{=}CH_2 + H_2O$	−44.1
17. $\mathbf{C_4H_4O} + 9/5CH_4 = 6/5CH_2{=}CHCH{=}CH_2 + CH_3OH$	−38.6
18. $\mathbf{C_4H_4O} + 2CH_4 = CH_2{=}CHCH{=}CH_2 + CH_3OCH_3$	−35.0
19. $\mathbf{C_4H_4O} + 4/3CH_3CH_3 = 5/3CH_2{=}CHCH{=}CH_2 + H_2O$	−44.6
20. $\mathbf{C_4H_4O} + 8CH_3OH = 3CH_2{=}CHCH{=}CH_2 + 9H_2O$	−87.8
21. $\mathbf{C_4H_4O} + 4CH_2{=}CH_2 = 3CH_2{=}CHCH{=}CH_2 + H_2O$	−47.5
22. $\mathbf{C_4H_4O} + 4CH_3OCH_3 = 3\ CH_2{=}CHCH{=}CH_2 + 5H_2O$	−80.3
23. $\mathbf{C_4H_4O} + 3/2CH_3CH_3 = 3/2CH_2{=}CHCH{=}CH_2 + CH_3OH$	−39.2
24. $\mathbf{C_4H_4O} + 5/3CH_3CH_3 = 4/3CH_2{=}CHCH{=}CH_2 + CH_3OCH_3$	−35.7
25. $\mathbf{C_4H_4O} + 9/2CH_2{=}CH_2 = 3CH_2{=}CHCH{=}CH_2 + CH_3OH$	−42.4
26. $\mathbf{C_4H_4O} + 9CH_3OCH_3 = 3CH_2{=}CHCH{=}CH_2 + 10CH_3OH$	−71.0
27. $\mathbf{C_4H_4O} + 5CH_2{=}CH_2 = 3CH_2{=}CHCH{=}CH_2 + CH_3OCH_3$	−39.3
average:	−35.9
std dev	21.8

(b) Isodesmic RERs

	$\Delta_f H^\circ_{298}(A_i)$
1. $\mathbf{C_4H_4O} + 4CH_4 + H_2O = CH_3CH_3 + 2CH_3OH + 2CH_2{=}CH_2$	−31.1
2. $\mathbf{C_4H_4O} + 2CH_4 + H_2O = 2CH_3OH + CH_2{=}CHCH{=}CH_2$	−33.2
3. $\mathbf{C_4H_4O} + 4CH_4 = CH_3CH_3 + 2CH_2{=}CH_2 + CH_3OCH_3$	−33.0
4. $\mathbf{C_4H_4O} + 2CH_4 = CH_3OCH_3 + CH_2{=}CHCH{=}CH_2$	−35.0
5. $\mathbf{C_4H_4O} + CH_3CH_3 + 2CH_2{=}CH_2 + H_2O = 2CH_3OH + 2\ CH_2{=}CHCH{=}CH_2$	−35.3
6. $\mathbf{C_4H_4O} + CH_3CH_3 + 2CH_2{=}CH_2 = CH_3OCH_3 + 2CH_2{=}CHCH{=}CH_2$	−37.1
average:	−34.2
std dev	2.1

(7) Benzene − C_6H_6

(a) Conventional RERs

	$\Delta_f H^\circ_{298}(A_i)$
1. $\mathbf{C_6H_6} + 12CH_4 = 9CH_3CH_3$	86.2
2. $\mathbf{C_6H_6} + 3CH_4 = 9/2CH_2{=}CH_2$	86.4
3. $\mathbf{C_6H_6} + 3CH_4 = 3CH_2{=}CHCH_3$	85.4
4. $\mathbf{C_6H_6} + 6/5CH_4 = 9/5CH_2{=}CHCH{=}CH_2$	82.6
5. $\mathbf{C_6H_6} + 3CH_3CH_3 = 6CH_2{=}CH_2$	86.5
6. $\mathbf{C_6H_6} + 3CH_3CH_3 = 4CH_2{=}CHCH_3$	85.2
7. $\mathbf{C_6H_6} + CH_3CH_3 = 2CH_2{=}CHCH{=}CH_2$	82.2
8. $\mathbf{C_6H_6} + 2CH_2{=}CHCH_3 = 4CH_2{=}CHCH{=}CH_2$	80.8
9. $\mathbf{C_6H_6} + 3CH_2{=}CH_2 = 3CH_2{=}CHCH{=}CH_2$	80.1
average:	83.9
std dev	2.5

(b) Isodesmic RERs

	$\Delta_f H^\circ_{298}(A_i)$
1. $\mathbf{C_6H_6} + 6CH_4 = 3CH_3CH_3 + 3CH_2{=}CH_2$	86.4
2. $\mathbf{C_6H_6} + 3CH_4 = 3CH_2{=}CHCH_3$	85.4
3. $\mathbf{C_6H_6} + 3CH_3CH_3 + 3CH_2{=}CH_2 = 6CH_2{=}CHCH_3$	84.5
4. $\mathbf{C_6H_6} + 3CH_4 = 3/2CH_3CH_3 + 3/2CH_2{=}CHCH_3$	83.2
5. $\mathbf{C_6H_6} + 3CH_2{=}CH_2 = 3CH_2{=}CHCH_3$	80.1
average:	83.9
std dev	2.4

[a] $\Delta_f H^\circ_{298}(A_i)$ (kJ/mol) is the standard enthalpy of formation of the species obtained using the conventional and isodesmic RERs at the G2 level.

stoichiometrically distinct. The average value of the standard enthalpy of formation of C_2H_4O obtained from these calculations coincides exactly with the value obtained from the conventional RERs.

We have also considered several other species (Table 1). Some of these are strained (cyclopropane, ethylene oxide, thiirane, aziridine, and cyclobutene) whereas others are aromatic (furan and benzene). Despite the diversity of the structural components of these molecules and the variety of sources of destabilization and stabilization, our suggested methodology more reliably and more simply reproduces the experimentally measured values for the enthalpies of formation of these substances while maintaining the calculational and conceptual framework of isodesmic reactions.

6. Concluding Remarks

From the stoichiometric analysis presented above, it is apparent that the arbitrariness of chemical reactions in a multiple chemical reaction system that is subject to additional bond-preserving constraints may be avoided. The reasoning that leads to a unique stoichiometric description is similar to that employed in systems that are subject only to mass-balance conditions. Thus, the bond-preserving conditions may be easily incorporated into the general RERs formalism.

In this work, we focused our attention mainly on the application of RERs in evaluating the enthalpy of formation of species from their total energies calculated using ab initio methods. Our preliminary analysis shows that both conventional and isodesmic RERs may be successfully used in converting the total energies of the species into accurate thermochemical data. Surprisingly, provided the set of reference species is the same in both types of RERs, we obtain almost the same accuracy of the thermochemistry. Clearly, this observation does not mean that the bond-preserving principle is not dominant in obtaining accurate thermochemical data. First, the systems analyzed in this work are rather exceptional in the sense that accurate experimental data exist for all of the reference species. Second, the fact that the conventional and isodesmic RERs give the same results in some cases far from justifies that this is a general result. Last, there exists a huge body of accumulated experience proving the validity of the bond-preserving principle.[12−17]

The newly defined isodesmic RERs may be fruitfully applied to solve a wide range of other problems in physical organic chemistry. In particular, the stabilization/destabilization effects in cyclic molecules may be better understood using a complete set of isodesmic RERs rather than single (and, in many cases, arbitrary) isodesmic reactions. Detailed applications of the isodesmic RERs to various physicochemical problems will be communicated in due course.

Finally, in complex systems, the generation and enumeration of RERs may become a real computational challenge because of their large number. In this respect, the development of the computer software that will generate and enumerate complete sets of both conventional and isodesmic RERs becomes crucial. Work along these lines is in progress.

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